1. Main Contributions
We focus on a ring-road mixed traffic system with one CAV and multiple heterogeneous human-driven vehicles (HDVs).

- **Controllability analysis**: we prove that the ring-road mixed traffic system is stabilizable under a mild condition.
- **Optimal controller synthesis**: we consider a system-level control objective and the communication topology.

2. System Modeling
Core idea: traffic system ⇔ network system

3. Controllability and Stabilizability

**Theorem 1**

1) System (1) is not completely controllable; there exists an uncontrollable mode corresponding to a zero eigenvalue.

2) If we have
\[ \alpha_{j1}^2 - \alpha_{i2} \alpha_{j1} \alpha_{j3} + \alpha_{i1} \alpha_{j2}^2 \neq 0, \; \forall i,j \in \{1,2,\ldots,n\}, \] (2)
then, system (1) is stabilizable.

**Proof:** PBH test + eigenvalue-eigenvector analysis

**Remark**
- The uncontrollable mode ⇒ the ring-road constraint
- To stabilize traffic flow at exactly \( v^* \), \( s_i^* = L - \sum_{i=2}^{n} s_i^* \)
- No requirement on system size \( n \) or the stability property of the original traffic system with HDVs only.
- The heterogeneous mixed traffic system (1) is stabilizable with probability one.

4. Optimal Controller Synthesis
Local-level (focus on CAVs only) ⇒ system-level (improve the entire traffic flow).

- Introduce disturbances \( \dot{x}(t) = Ax(t) + Bu(t) + Hu(t) \)
- System-level output
\[ u(t) = [Q^\frac{1}{2}, 0]^T x(t) + [0, R^\frac{1}{2}]^T u(t) \]

The pre-specified communication topology is imposed as a **structured constraint** on feedback gain: \( K \in \mathcal{K} \).

- Sparsity invariance: \( Z \in \mathcal{T}, X \in \mathcal{S} \Rightarrow K = ZX^{-1} \in \mathcal{K} \).
- Convex relaxation formulation

\[
\begin{align*}
\min_{X,Y,Z} & \quad \text{Trace}(QX) + \text{Trace}(RY) \\
\text{s.t.} & \quad AX + XA^T - BZ - Z^TB^T + HH^T \leq 0, \\
& \quad Y \geq 0, \; X > 0, \; Z \in \mathcal{T}, \; X \in \mathcal{S}
\end{align*}
\]

5. Numerical Experiments & Conclusions

**Conclusion 1**: one single CAV can stabilize traffic flow, and can also regulate traffic flow to a higher or lower velocity.

**Conclusion 2**: the proposed controller enables one single CAV with limited communication ability to dampen traffic waves actively.